Paper Reference(s)

6668/01

Edexcel GCE

Further Pure Mathematics FP2

Advanced Level

Thursday 23 June 2011 – Morning

Time: 1 hour 30 minutes

<u>**Materials required for examination**</u>	<u>**Items included with question papers**</u>
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP2), the paper reference (6668), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P38168A

1. Find the set of values of x for which

$$\frac{3}{x+3} > \frac{x-4}{x}.$$

(7)

2.
$$\frac{d^2y}{dx^2} = e^x\left(2y\frac{dy}{dx} + y^2 + 1\right).$$

(*a*) Show that

$$\frac{d^3y}{dx^3} = e^x\left[2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + ky\frac{dy}{dx} + y^2 + 1\right],$$

where k is a constant to be found.

(3)

Given that, at $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$,

(*b*) find a series solution for y in ascending powers of x, up to and including the term in x^3.

(4)

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3. Find the general solution of the differential equation

$$x\frac{dy}{dx} + 5y = \frac{\ln x}{x}, \quad x > 0,$$

giving your answer in the form $y = f(x)$.

(8)

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4. Given that

$$(2r+1)^3 = Ar^3 + Br^2 + Cr + 1,$$

(*a*) find the values of the constants A, B and C.

(2)

(*b*) Show that

$$(2r+1)^3 - (2r-1)^3 = 24r^2 + 2.$$

(2)

(*c*) Using the result in part (*b*) and the method of differences, show that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$

(5)

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5. The point P represents the complex number z on an Argand diagram, where

$$|z - \mathrm{i}| = 2.$$

The locus of P as z varies is the curve C.

(*a*) Find a cartesian equation of C.

(2)

(*b*) Sketch the curve C.

(2)

A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z + \mathrm{i}}{3 + \mathrm{i}z}, \quad z \neq 3\mathrm{i}.$$

The point Q is mapped by T onto the point R. Given that R lies on the real axis,

(*c*) show that Q lies on C.

(5)

6.

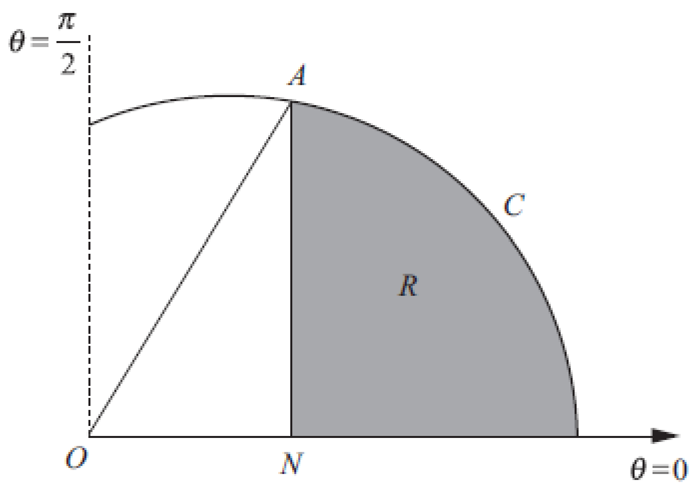

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos\theta, \qquad 0 \leq \theta \leq \frac{\pi}{2}.$$

At the point A on C, the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R.

(9)

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7. (*a*) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$$

(5)

Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$,

(*b*) find all the solutions of

$$\sin 5\theta = 5\sin 3\theta,$$

in the interval $0 \leq \theta < 2\pi$. Give your answers to 3 decimal places.

(6)

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8. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = \cos 3t, \qquad t \geq 0,$$

describes the motion of a particle along the x-axis.

(*a*) Find the general solution of this differential equation.

(8)

(*b*) Find the particular solution of this differential equation for which, at $t = 0$, $x = \frac{1}{2}$ and $\frac{dx}{dt} = 0$.

(5)

On the graph of the particular solution defined in part (*b*), the first turning point for $t > 30$ is the point A.

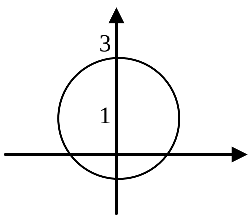
(*c*) Find approximate values for the coordinates of A.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$3x=(x-4)(x+3) \qquad x^2-4x-12=0$	M1
	$x=-2,\ x=6$ both	A1
	Other critical values are $x=-3,\ x=0$	B1, B1
	$-3<x<-2, \qquad 0<x<6$	M1 A1 A1
		(7) **7**
2. **(a)**	$\dfrac{d^3y}{dx^3}=e^x\left(2y\dfrac{d^2y}{dx^2}+2\left(\dfrac{dy}{dx}\right)^2+2y\dfrac{dy}{dx}\right)+e^x\left(2y\dfrac{dy}{dx}+y^2+1\right)$	M1 A1
	$\dfrac{d^3y}{dx^3}=e^x\left(2y\dfrac{d^2y}{dx^2}+2\left(\dfrac{dy}{dx}\right)^2+4y\dfrac{dy}{dx}+y^2+1\right) \qquad (k=4)$	A1
		(3)
(b)	$\left(\dfrac{d^2y}{dx^2}\right)_0=e^0\left(4+1+1\right)=6$	B1
	$\left(\dfrac{d^3y}{dx^3}\right)_0=e^0\left(12+8+8+1+1\right)=30$	B1
	$y=1+2x+\dfrac{6x^2}{2}+\dfrac{30x^3}{6}=1+2x+3x^2+5x^3$	M1 A1ft
		(4) **7**
3.	$\dfrac{dy}{dx}+5\dfrac{y}{x}=\dfrac{\ln x}{x^2} \qquad$ Integrating factor $e^{\int\frac{5}{x}}$	M1
	$e^{\int\frac{5}{x}}=e^{5\ln x}=x^5$	A1
	$\int x^3\ln x\,dx=\dfrac{x^4\ln x}{4}-\int\dfrac{x^3}{4}dx$	M1 M1 A1
	$=\dfrac{x^4\ln x}{4}-\dfrac{x^4}{16} \quad (+C)$	A1
	$x^5y=\dfrac{x^4\ln x}{4}-\dfrac{x^4}{16}+C \qquad y=\dfrac{\ln x}{4x}-\dfrac{1}{16x}+\dfrac{C}{x^5}$	M1 A1
		(8) **8**

Question Number	Scheme	Marks
4. **(a)**	$(2r+1)^3=(2r)^3+3(2r)^2+3(2r)+1$ $A=8,\ B=12,\ C=6$	M1 A1 (2)
(b)	$(2r-1)^3=(2r)^3-3(2r)^2+3(2r)-1$ $(2r+1)^3-(2r-1)^3=24r^2+2$ (*)	M1 A1cso (2)
(c)	$r=1:\quad 3^3-1^3 \quad = \quad 24\times1^2+2$ $r=2:\quad 5^3-3^3 \quad = \quad 24\times2^2+2$ $\vdots \qquad\qquad \vdots$ $r=n:\quad (2n+1)^3-(2n-1)^3 \quad = \quad 24\times n^2+2$ Summing: $(2n+1)^3-1=24\sum r^2+\left(\sum\ \right)2$ $\left(\sum 2\right)=2n$ Proceeding to $\sum_{r=1}^{n} r^2=\frac{1}{6}n(n+1)(2n+1)$	M1 A1 M1 B1 A1cso (5) **9**
5. **(a)**	$x^2+(y-1)^2=4$	M1 A1 (2)
(b)	 M1: Sketch of circle A1: Evidence of correct centre and radius	M1 A1 (2)
(c)	$w=\frac{(x+\mathrm{i}y)+\mathrm{i}}{3+\mathrm{i}(x+\mathrm{i}y)}=\frac{x+\mathrm{i}(y+1)}{(3-y)+\mathrm{i}x}$ $=\frac{[x+\mathrm{i}(y+1)][(3-y)-\mathrm{i}x]}{[(3-y)+\mathrm{i}x][(3-y)-\mathrm{i}x]}$ On x-axis, so imaginary part = 0: $(y+1)(3-y)-x^2=0$ $(y+1)(3-y)-x^2=0 \quad\Rightarrow\quad x^2+(y-1)^2=4$, so Q is on C	M1 M1 M1 A1 A1cso (5) **9**

Question Number	Scheme	Marks
6.	$2+\cos\theta=\frac{5}{2}\Rightarrow\theta=\frac{\pi}{3}$	B1
	$\frac{1}{2}\int(2+\cos\theta)^2d\theta=\frac{1}{2}\int\left(4+4\cos\theta+\cos^2\theta\right)d\theta$	M1
	$=\frac{1}{2}\left[4\theta+4\sin\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}\right]$	M1 A1
	Substituting limits $\left(\frac{1}{2}\left[\frac{9\pi}{6}+4\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{8}\right]=\frac{1}{2}\left(\frac{3\pi}{2}+\frac{17\sqrt{3}}{8}\right)\right)$	M1
	Area of triangle = $\frac{1}{2}(r\cos\theta)(r\sin\theta)=\frac{1}{2}\times\frac{25}{4}\times\frac{1}{2}\times\frac{\sqrt{3}}{2}\left(=\frac{25\sqrt{3}}{32}\right)$	M1 A1
	Area of R = $\frac{3\pi}{4}+\frac{17\sqrt{3}}{16}-\frac{25\sqrt{3}}{32}=\frac{3\pi}{4}+\frac{9\sqrt{3}}{32}$	M1 A1
		(9) **9**
7. **(a)**	$\sin 5\theta=\text{Im}(\cos\theta+\text{i}\sin\theta)^5$	B1
	$5\cos^4\theta(\text{i}\sin\theta)+10\cos^2\theta(\text{i}^3\sin^3\theta)+\text{i}^5\sin^5\theta$	M1
	$=\text{i}(5\cos^4\theta\sin\theta-10\cos^2\theta\sin^3\theta+\sin^5\theta)$	A1
	$\left(\text{Im}(\cos\theta+\text{i}\sin\theta)^5\right)=5\sin\theta(1-\sin^2\theta)^2-10\sin^3\theta(1-\sin^2\theta)+\sin^5\theta$	M1
	$\sin 5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ (*)	A1cso
		(5)
(b)	$16\sin^5\theta-20\sin^3\theta+5\sin\theta=5\left(3\sin\theta-4\sin^3\theta\right)$	M1
	$16\sin^5\theta-10\sin\theta=0$	M1
	$\sin^4\theta=\frac{5}{8}$ $\theta=1.095$	A1
	Inclusion of solutions from $\sin\theta=-\sqrt[4]{\frac{5}{8}}$	M1
	Other solutions: $\theta=2.046,\ 4.237,\ 5.188$	A1
	$\sin\theta=0\Rightarrow\theta=0,\ \theta=\pi$ (3.142)	B1
		(6) **11**

Question Number	Scheme	Marks
8.		
(a)	$m^2+6m+9=0 \qquad m=-3$	M1
	C.F. $\quad x=\left(A+Bt\right)e^{-3t}$	A1
	P.I. $\quad x=P\cos 3t+Q\sin 3t$	B1
	$\dot{x}=-3P\sin 3t+3Q\cos 3t$ $\ddot{x}=-9P\cos 3t-9Q\sin 3t$	M1
	$\left(-9P\cos 3t-9Q\sin 3t\right)+6\left(-3P\sin 3t+3Q\cos 3t\right)+9\left(P\cos 3t+Q\sin 3t\right)=\cos 3t$	M1
	$-9P+18Q+9P=1 \quad$ and $\quad -9Q-18P+9Q=0$	M1
	$P=0 \quad$ and $\quad Q=\dfrac{1}{18}$	A1
	$x=(A+Bt)e^{-3t}+\dfrac{1}{18}\sin 3t$	A1ft
		(8)
(b)	$t=0: \quad x=A=\dfrac{1}{2}$	B1
	$\dot{x}=-3(A+Bt)\mathrm{e}^{-3t}+B\mathrm{e}^{-3t}+\dfrac{3}{18}\cos 3t$	M1
	$t=0: \quad \dot{x}=-3A+B+\dfrac{1}{6}=0 \qquad B=\dfrac{4}{3}$	M1 A1
	$x=\left(\dfrac{1}{2}+\dfrac{4t}{3}\right)\mathrm{e}^{-3t}+\dfrac{1}{18}\sin 3t$	A1
		(5)
(c)	$t\approx\dfrac{59\pi}{6} \quad (\approx 30.9)$	B1
	$x\approx-\dfrac{1}{18}$	B1ft
		(2) **15**